$SU(N)$ symmetry implies nonzero vacuum expectation value $\langle \Phi_F \rangle = \sqrt{N}(f + 0(1/N))$ and the associated $N-1$ Goldstone bosons (with $\Phi_\perp(x)$ as the interpolating fields) whereas the N^{th} "flavoured" scalar boson corresponding to $\Phi_F(x)$ is, "confined" (6.b) (cf. [11] for the case of LT phase in the usual $0(N)$ nonlinear sigma model). The Higgs mechanism adds a "mass" term $2|f|^2$ to the denominator of $\langle A_\mu A_\nu \rangle^{(0)}$ (6.b), but no particle pole is created, i. e. the "massive photon" is absent from the spectrum. The present LT phase is to be contrasted with the conventional (four dimensional) Abelian Higgs model [12], where there are massive vector and scalar mesons corresponding to $J_\mu(x)$ (the gauge invariant current) and to (the gauge invariant generalization of) Φ_F, resp.

The expressions (6.c) show that the free propagators acquire a scale invariant form in the critical CP^{N-1} theory (i. e. $e^{-2} = \lambda^{-1} \equiv 0$). The exact scale invariance of the full gauge invariant universal theory at the critical point (with anomalous dimension for the Φ field $\zeta_\Phi = \eta/2$) is proved rigorously in [7], taking into account the UV and IR renormalization. In the leading order of $1/N$ we find for the basic critical exponent η: $\eta^{(1)} = -20/N\pi^2$ (the minus sign is connected with the lack of positivity in the "big" Hilbert space).

The existence of a phase transition in models (1) and (2) was announced in the third ref. [6]. The particle spectrum of (2) which here is implied by the explicitly found $1/N$ graphical rules (6) was previously known to I. Ya. Aref'eva and S. Azakov (private communication to the second named author).

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PHASE TRANSITION AND PARTICLE SPECTRUM IN THREE DIMENSIONAL GENERALIZED NONLINEAR SIGMA MODELS AND HIGGS MODELS FROM 1/N EXPANSION

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Generalized nonlinear sigma models (GNLSM) (CP^{N-1} [1–3], $G_{N,n}$ [1,4-6]) have recently attracted much interest, because of their striking similarities with realistic gauge theories (instantons, asymptotic freedom, confinement). In this note (and in a subsequent one [7]) we shall consider the GNLSMs and the Higgs models with internal SU(N) "flavour" symmetry in $D=3$ Euclidean space-time dimensions in the context of critical phenomena and universality. As shown here (at the instance of CP^{N-1} and the Abelian Higgs models for the sake of simplicity) that these models exhibit a second order phase transition through the same mechanism as in the usual $O(N)$ (non)linear sigma model [8]. The graphical rules of the $1/N$ expansion and the particle spectrum of the arising phases are described. In [7] we shall apply the "soft mass" renormalization procedure [9, 10] to prove the ultraviolet(UV) and infrared(IR) renormalizability of the CP^{N-1} model and the fact that the latter is an IR fixed point universal theory of the noncanonically renormalized Abelian Higgs model.

The Lagrangians of the models in question can be written in the following form:

$$(1)\quad L^{(1)}(x) = -(\nabla_\nu \Phi)^*(\nabla_\nu \Phi) - \sigma'(\Phi^*\Phi - N/T\mu) + \frac{N}{2\lambda\mu}\sigma'^2 - \frac{N}{4e^2\mu}F^2_{\lambda\nu} - iNB\partial_\nu A_\nu,$$

$$(2)\quad L^{(2)}(x) = -(\nabla_\nu z)^*(\nabla_\nu z) - \sigma'(z^*z - N/T\mu) - iNB\partial_\nu A_\nu,$$

$$\nabla_\nu = \partial_\nu + iA_\nu,\quad F_{\lambda\nu} = \partial_\lambda A_\nu - \partial_\nu A_\lambda,\quad z^*z = \sum_{a=1}^{N} z^*_a z_a,\quad \Phi^*\Phi \equiv \sum_{a=1}^{N} \Phi^*_a \Phi_a \text{ etc.},$$

where λ and e^2 are dimensionless coupling constants, T-temperature, μ — mass scale parameter; $\sigma'(x) = \sigma_0 + i\sigma(x)$ (σ_0 — arbitrary but fixed nonnegative constant) and $B(x)$ — auxiliary fields the latter enforcing the Landau gauge. Formally (2) is obtained from (1) in the strong coupling limit.

The generating functional for the Euclidean Green's functions of (1) or (2) reads (the chiral and Higgs fields will be denoted henceforth by a common letter: $\Phi(x)$]:

$$W[J, J^*, K]=\int\prod_x d\Phi d\Phi^* d\sigma' dA_\mu dB \exp\left\{\int d^3x[L^{(\cdot)}(x)+J^*\Phi+\Phi^*J+K_\mu A_\mu]\right\} \tag{3}$$
$$=\int\prod_x d\Phi_F d\Phi_F^* d\sigma' dA_\mu dB \exp\{NS_1+S_2\};$$

$$S_1=-\mathrm{Tr}\ln(-\nabla_\nu\nabla_\nu+\sigma')+\int d^3x[-(\nabla_\nu\Phi_F)^*(\nabla_\nu\Phi_F)-\sigma'(\Phi_F^*\Phi_F-\mu/T)$$
$$+1/2\lambda\mu\,\sigma'^2-1/4e^2\mu\,F^2_{\lambda\nu}-iB\partial_\nu A_\nu], \tag{4}$$
$$S_2=\mathrm{Tr}\ln(-\nabla_\nu\nabla_\nu+\sigma')+\int d^3x[K_\nu A_\nu+\sqrt{N}(J_F^*\Phi_F+\Phi_F^*J_F)]$$
$$+\int d^3x\,d^3y\,J_\perp^*(x)[-\nabla_\nu\nabla_\nu+\sigma']^{-1}(x,y)J_\perp(y),$$
$$\Phi=\Phi_\perp+\sqrt{N}\Phi_F\widehat{F},\ J=J_\perp+J_F\widehat{F},\ \sum_{a=1}^{N}\Phi^*_{\perp a}\widehat{F}_a=\sum_{a=1}^{N}J^*_{\perp a}\widehat{F}_a=0.$$

Here F is an arbitrary fixed complex unit vector in "flavour" space. To arrive at the last equality in (3) we have performed the Gaussian integration over $\Phi_\perp$, $\Phi^*_\perp$ [8,11]. The $1/N$ expansion of (3) is equivalent to the expansion around constant saddle points of the action $S_1(4)$: $\sigma'^{(c)}=\sigma_0\equiv m^2$, $\Phi_F^{(c)}=f$, $A_\nu^{(c)}=B^{(c)}=0$. The equations for determination of the latter are exactly the same as for the usual $U(N)$ (non)linear sigma model:

$$\int\frac{d^3k}{(2\pi)^3}[m^2+k^2]^{-1}-\mu/T+|f|^2-m^2/\lambda\mu=0,\quad m^2f=m^2f^*=0. \tag{5}$$

The first (mass gap) eq. (5) can be renormalized by e. g. a "soft mass" subtraction in the divergent integral:

$$\int\frac{d^3k}{(2\pi)^3}[(m^2+k^2)^{-1}-(\mu^2+k^2)^{-1}]+a_0\mu/4\pi=\mu/T_c-m/4\pi,\quad T_c\equiv 4\pi(1+a_0)^{-1},$$

a_0 being an arbitrary dimensionless constant accounting for the subtraction ambiguity. Thus we obtain three types of solutions of eqs. (5):

a) High temperature (HT) phase solution: $T>T_c$, $f=0$, $m=4\pi\mu\left(\frac{1}{T_c}-\frac{1}{T}\right)$ for the CP^{N-1} model, $m=-\lambda\mu/8\pi+\left[(\lambda\mu/8\pi)^2+\lambda\mu^2\left(\frac{1}{T_c}-\frac{1}{T}\right)\right]^{1/2}$ for (1).

b) Low temperature (LT) phase solution: $T<T_c$, $m=0$, $|f|^2=\mu\left(\frac{1}{T}-\frac{1}{T_c}\right)$.

c) Critical theory (for (2)) or preasymptotic zero mass theory (for (1)), resp.: $T=T_c$, $m=f=0$.

The free propagators in momentum space for the cases (a), (b) and (c) take the following form (see Fig. 1; t^0 denotes the zero order Taylor subtraction operator in p, m around $p=0$, $m=\mu$):

$$\langle\Phi_a\Phi_b^*\rangle^{(0)}=\delta_{ab}(m^2+p^2)^{-1},\ \langle\sigma\sigma\rangle^{(0)}=N^{-1}[(4\pi|p|)^{-1}\mathrm{arctg}(|p|/2m)+1/\lambda\mu]^{-1},$$
$$\langle A_\mu A_\nu\rangle^{(0)}=N^{-1}(\delta_{\mu\nu}-p_\mu p_\nu/p^2)H^{-1}(p^2) \tag{6.a}$$
$$H(p^2)\equiv p^2/\mu e^2+(4m^2+p^2)(8\pi|p|)^{-1}\mathrm{arctg}(|p|/2m)-m/4\pi;$$
$$\langle\Phi_{\perp a}\Phi^*_{\perp b}\rangle^{(0)}=(\delta_{ab}-\widehat{F}_a\widehat{F}_b)1/p^2,\ \langle\Phi_F\Phi_F^*\rangle^{(0)}=(p^2+8|f|^2|p|)^{-1},$$
$$\langle\sigma\sigma\rangle^{(0)}=N^{-1}p^2(|p|/8+|f|^2+p^2/\lambda\mu)^{-1}$$

$$\langle A_\mu A_\nu\rangle^{(0)}=N^{-1}(\delta_{\mu\nu}-p_\mu p_\nu/p^2)[p^2/\mu e^2+2|f|^2+|p|/16]^{-1}; \tag{6.b}$$
$$\langle\Phi_a\Phi_b^*\rangle^{(0)}=\delta_{ab}/p^2,\ \langle\sigma\sigma\rangle^{(0)}=N^{-1}8|p|(1+8|p|/\lambda\mu)^{-1}.$$
$$\langle A_\mu A_\nu\rangle^{(0)}=N^{-1}(\delta_{\mu\nu}-p_\mu p_\nu/p^2)(p^2/\mu e^2+|p|/16)^{-1}. \tag{6.c}$$

In all three cases the mixed propagators are the same: $\langle A_\mu B\rangle=-\langle BA_\mu\rangle=p_\mu/Np^2$, and $\langle BB\rangle=0$. In Fig. 2 the so called "forbidden" subgraphs are depicted.

The particle spectrum can now easily be read off (6) (as long as the $1/N$ expansion is valid). In both phases there is no (bound state) particle corresponding to the σ field. In the HT phase (also called QED phase), there are a $SU(N)$ N-plet of "flavoured" massive scalar bosons (for (2) the mass is dynamically generated) and "photon" because $\langle A_\mu A_\nu\rangle^{(0)}$ contains a massless pole: $H(p^2)\approx p^2(1/24\pi m+1/\mu e^2)$ for $p^2\to 0$.

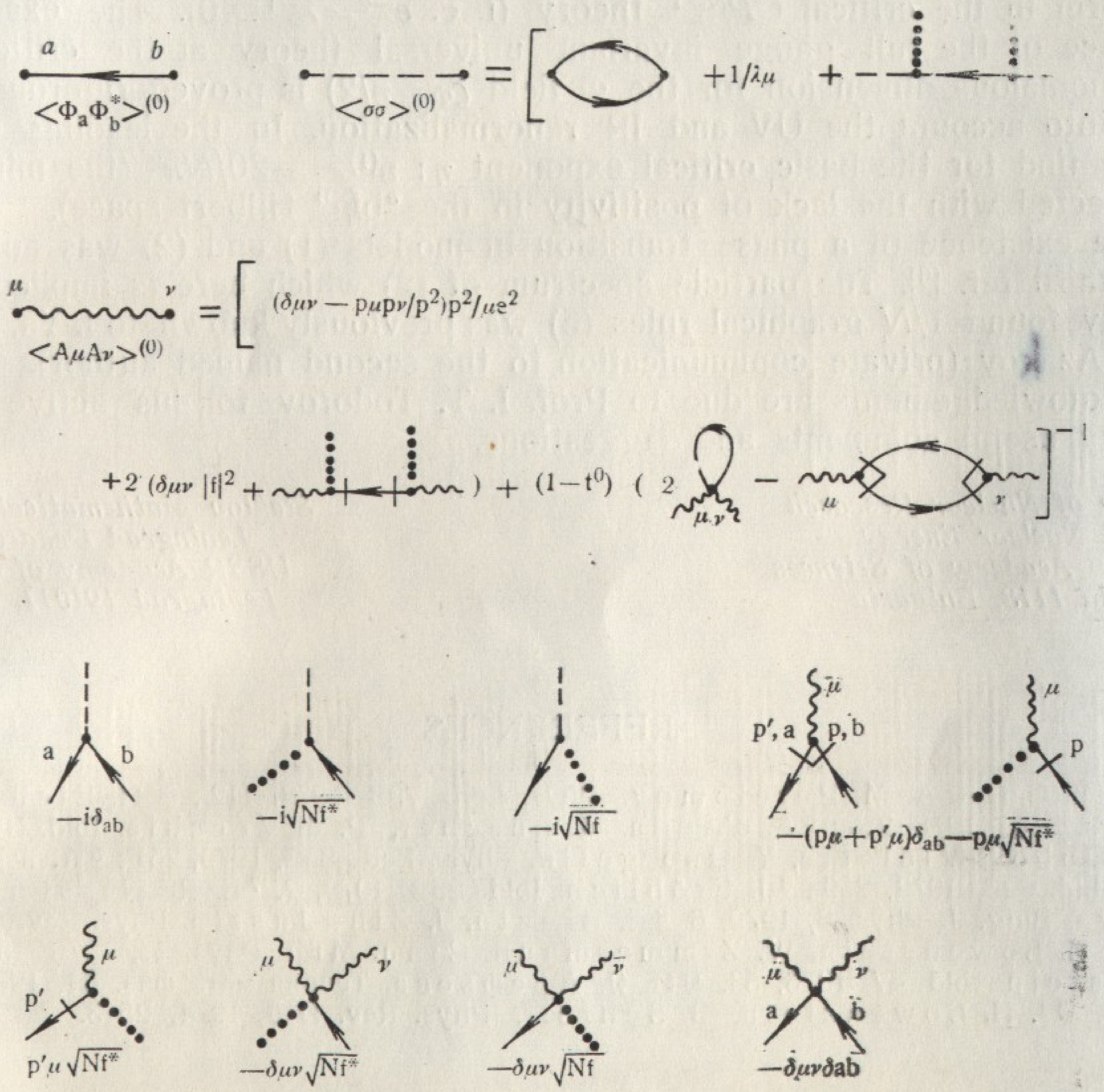

Fig. 1. Graphical elements of the $1/N$ expansion

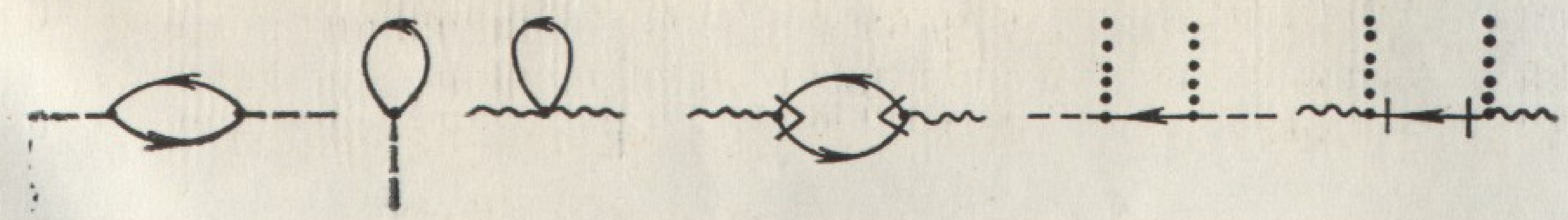

Fig. 2. "Forbidden" subgraphs

An interplay between the Higgs and the Goldstone phenomena takes place in the LT (Higgs-Goldstone) phase. The spontaneous breakdown of the global